Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 3 (6669/01)

Question Number	Scheme		Marks
	Mark (a) and (b) together		
1. (a) & (b)	$ae = 13$ **and** $a^2(e^2 - 1) = 25$	Sight of **both** of these (can be implied by their work) (allow $\pm$ ae $= \pm 13$ or $\pm$ ae $= 13$ or ae $= \pm 13$)	B1
	Solves to obtain $a^2 = \ldots$ or $a = \ldots$	Eliminates *e* to reach $a^2 = \ldots$ or $a = \ldots$	M1
	$a = 12$	Cao (not ± 12) unless -12 is rejected	A1
	$e = 13/$ "12"	Uses their *a* to find *e* or finds *e* by eliminating *a* (Ignore ± here) (Can be implied by a correct answer)	M1
	$x = (\pm)\frac{a}{e}, = \pm\frac{144}{13}$	M1: $(x =)(\pm)\frac{a}{e}$ $\pm$ not needed for this mark nor is *x* and even allow y = $(\pm)\frac{a}{e}$ here – just look for use of $\frac{a}{e}$ with numerical *a* and *e*. A1: $x = \pm\frac{144}{13}$ oe but must be an <u>equation</u> (Do not allow $x = \pm\frac{12}{13/12}$)	M1, A1
			Total 6
	If they use the eccentricity equation for the ellipse ($b^2=a^2(1-e^2)$) allow the M's		

Question Number	Scheme	Marks
2. (a)	$k\,\text{arsinh}\left(\frac{2x}{3}\right)$ $(+c)$ or $k\ln[px+\sqrt{(p^2x^2+\frac{9}{4}p^2)}]$ $(+c)$	M1
	$\frac{1}{2}\text{ar}\sinh\left(\frac{2x}{3}\right)$ $(+c)$ or $\frac{1}{2}\ln[px+\sqrt{(p^2x^2+\frac{9}{4}p^2)}]$ $(+c)$	A1
		(2)
(b)	So: $\frac{1}{2}\ln\left[6+\sqrt{45}\right]-\frac{1}{2}\ln\left[-6+\sqrt{45}\right]=\frac{1}{2}\ln\left[\frac{6+\sqrt{45}}{-6+\sqrt{45}}\right]$	M1
	Uses correct limits and combines logs	
	$=\frac{1}{2}\ln\left[\frac{6+\sqrt{45}}{-6+\sqrt{45}}\right]\left[\frac{6+\sqrt{45}}{6+\sqrt{45}}\right]=\frac{1}{2}\ln\left[\frac{(6+\sqrt{45})^2}{9}\right]$	M1
	Correct method to rationalise denominator (may be implied) **Method must be clear if answer does not follow their fraction**	
	$=\ln[2+\sqrt{5}]$ (or $\frac{1}{2}\ln[9+4\sqrt{5}]$)	A1cso
	Note that the last 3 marks can be scored without the need to rationalise e.g. $2\times\frac{1}{2}\left[\ln[2x+\sqrt{(4x^2+9)}]\right]_0^3=\ln(6+\sqrt{45})-\ln 3=\ln(\frac{6+\sqrt{45}}{3})$ M1: Uses the limits 0 and 3 and doubles M1: Combines Logs A1: $\ln[2+\sqrt{5}]$ oe	
		(3)
		Total 5
Alternative for **(a)**	$x=\frac{3}{2}\sinh u\Rightarrow\int\frac{1}{\sqrt{9\sinh^2u+9}}.\frac{3}{2}\cosh u\,\mathrm{d}u=k\,\text{arsinh}\left(\frac{2x}{3}\right)$ $(+c)$	M1
	$\frac{1}{2}\text{ar}\sinh\left(\frac{2x}{3}\right)$ $(+c)$	A1
Alternative for **(b)**	$\left[\frac{1}{2}\text{arsinh}\left(\frac{2x}{3}\right)\right]_{-3}^{3}=\frac{1}{2}\text{arsinh}\ \ 2\ -\frac{1}{2}\text{arsinh}\ \ -2$	
	$\frac{1}{2}\ln(2+\sqrt{5})-\frac{1}{2}\ln(\sqrt{5}-2)=\frac{1}{2}\ln(\frac{2+\sqrt{5}}{\sqrt{5}-2})$	M1
	Uses correct limits and combines logs	
	$=\frac{1}{2}\ln(\frac{2+\sqrt{5}}{\sqrt{5}-2}.\frac{\sqrt{5}+2}{\sqrt{5}+2})=\frac{1}{2}\ln(\frac{2\sqrt{5}+4+5+2\sqrt{5}}{5-4})$	M1
	Correct method to rationalise denominator (may be implied) **Method must be clear if answer does not follow their fraction**	
	$=\frac{1}{2}\ln[9+4\sqrt{5}]$	A1cso

Question Number	Scheme		Marks
3.	$(\frac{dx}{d\theta}) = 2\sinh 2\theta$ **and** $(\frac{dy}{d\theta}) = 4\cosh\theta$ Or equivalent correct derivatives		B1
	$A = (2\pi)\int 4\sinh\theta\sqrt{"2\sinh 2\theta"^2 + "4\cosh\theta"^2}\,d\theta$ or $A = (2\pi)\int 4\sinh\theta\sqrt{(1+(\frac{"4\cosh\theta"}{"2\sinh 2\theta"})^2}\,.2\sinh 2\theta\,d\theta$		M1
	Use of correct formula including replacing d*x* with "$2\sinh 2\theta$"$d\theta$ if chain rule used. Allow the omission of the 2π here.		
	$A = 32\pi\int \sinh\theta\cosh^2\theta\,d\theta$ $A = 32\pi\int (\sinh\theta + \sinh^3\theta)\,d\theta$		B1
	<u>Completely correct</u> expression for *A* with the square root removed **This mark may be recovered later if the 2π is introduced later**		
	$A = \frac{32\pi}{3}\left[\cosh^3\theta\right]_0^1$	M1: Valid attempt to integrate a correct expression or a multiple of a correct expression – dependent on the first M1 A1: Correct expression	dM1A1
	$= \frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	M1: Uses the limits 0 and 1 correctly. Dependent on **both** previous M"s A1: Cao and cso (no errors seen)	ddM1A1
			(7)
	Example Alternative Integration for last 4 marks		
	$\int \sinh\theta\cosh^2\theta\,d\theta = \int \sinh\theta(1+\sinh^2\theta)\,d\theta = \int(\sinh\theta + \sinh^3\theta)\,d\theta$ $\int(\sinh\theta + \frac{1}{4}\sinh 3\theta - \frac{3}{4}\sinh\theta)\,d\theta = \frac{1}{4}\int(\sinh\theta + \sinh 3\theta)\,d\theta$ $= \frac{1}{4}\cosh\theta + \frac{1}{12}\cosh 3\theta$ **dM1:** $\int \sinh\theta\cosh^2\theta\,d\theta = p\cosh\theta + q\cosh 3\theta$ **A1**: $32\pi\left[\frac{1}{4}\cosh\theta + \frac{1}{12}\cosh 3\theta\right]$		dM1A1
	$A = 8\pi\left[\cosh\theta + \frac{1}{3}\cosh 3\theta\right]_0^1$ $= 8\pi(\cosh 1 + \frac{1}{3}\cosh 3 - \cosh 0 - \frac{1}{3}\cosh 0)$ .... $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	M1: Uses the limits 0 and 1 correctly. Dependent on **both** previous M"s A1: Cao	ddM1A1

Question Number	Scheme		Marks
3.	**Alternative Cartesian Approach**		
	$x=1+\frac{y^2}{8}$	Any correct Cartesian equation	B1
	$\frac{dx}{dy}=\frac{y}{4}$ or $\frac{dy}{dx}=\frac{\sqrt{2}}{(x-1)^{\frac{1}{2}}}$	Correct Derivative	B1
	$A=\int 2\pi .y\sqrt{(1+\left(\frac{y}{4}\right)^2}) \mathrm{d}y$ or $A=\int 2\pi .\sqrt{8}(x-1)^{\frac{1}{2}}\sqrt{(1+\left(\frac{2}{x-1}\right)}) \mathrm{d}x$		M1
	Use of a correct formula		
	$A=2\pi\times\frac{2}{3}\times 8\left(1+\frac{y^2}{16}\right)^{\frac{3}{2}}$ or $A=\frac{4\pi\sqrt{8}}{3}\ \ x+1\ ^{\frac{3}{2}}$		dM1 A1
	M1: Convincing attempt to integrate a relevant expression – dependent on the first M1 but allow the omission of 2π		
	A1: Completely correct expression for A		
	$A=2\pi\times\frac{2}{3}\times 8\ \ 1+\sinh^2 1\ ^{\frac{3}{2}}-2\pi\times\frac{2}{3}\times 8$ or $2\pi\times\frac{2}{3}\times\sqrt{8}\ \ 1+\cosh 2\ ^{\frac{3}{2}}-\frac{32\pi}{3}$		ddM1
	Correct use of limits (0 → 4sinh1 for y or 1 → cosh2 for *x*)		
	Use $1+\sinh^2 1=\cosh^2 1$ to give $\frac{32\pi}{3}\left[\cosh^3 1-1\right]$	Use $\cosh 2=2\cosh^2 1-1$ to give $\frac{32\pi}{3}\left[\cosh^3 1-1\right]$	A1

Question Number	Scheme		Marks
4.	$\frac{dy}{dx} = \frac{40}{\sqrt{(x^2-1)}}$ - 9	M1: $\frac{dy}{dx} = \frac{p}{\sqrt{(x^2-1)}} - q$ A1: Cao	M1 A1
	Put $\frac{dy}{dx}$ **= 0** and obtain $x^2 = ...$ **(Allow sign errors only)**	e.g. $\left(\frac{1681}{81}\right)$	dM1
	$x = \frac{41}{9}$	M1: Square root A1: $x = \frac{41}{9}$ or exact equivalent (not $\pm\frac{41}{9}$)	M1 A1
	$y = 40\ln\{(\frac{41}{9}) + \sqrt{(\frac{41}{9})^2 - 1}\} - "41"$	Substitutes $x = "\frac{41}{9}"$ into the curve and uses the logarithmic form of arcosh	M1
	So $y = 80\ln 3$ - 41	Cao	A1
			Total 7

Question Number	Scheme		Marks
5. (a) (i)&(ii)	$\begin{pmatrix}1 & 1 & a\\ 2 & b & c\\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}1+a\\ b+c\\ 1\end{pmatrix}=\lambda_1\begin{pmatrix}0\\1\\1\end{pmatrix}$, and so a = -1, λ_1 = 1		M1, A1, A1
	M1: Multiplies out matrix with first eigenvector and puts equal to λ_1 times eigenvector. A1 : Deduces *a* = -1. A1: Deduces λ_1 = 1		
	$\begin{pmatrix}1 & 1 & a\\ 2 & b & c\\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}1-a\\ 2-c\\ -2\end{pmatrix}=\lambda_2\begin{pmatrix}1\\0\\-1\end{pmatrix}$, and so $c = 2,\ \lambda_2 = 2$		M1, A1, A1
	M1: Multiplies out matrix with second eigenvector and puts equal to λ_2 times eigenvector. A1: Deduces *c* = 2. A1: Deduces λ_2 = 2		
	$b+c=\lambda_1$ so b = -1	M1: Uses $b+c=\lambda_1$ with their λ_1 to find a value for *b* (They must have an equation in *b* and *c* from the first eigenvector to score this mark) A1: *b* = -1	M1A1
	(a = -1, b = -1, c = 2, λ_1=1, λ_2 = 2)		(8)
(b)(i)	detP = -*d* - 1	Allow 1 – *d* – 2 or 1 – (2 + *d*) A correct (possibly un-simplified) determinant	B1
(ii)	$\mathbf{P}^T=\begin{pmatrix}1 & 2 & -1\\ 1 & 1 & 0\\ 0 & d & 1\end{pmatrix}$ **or** minors $\begin{pmatrix}1 & d+2 & 1\\ 1 & 1 & 1\\ d & d & -1\end{pmatrix}$ **or** cofactors $\begin{pmatrix}1 & -2-d & 1\\ -1 & 1 & -1\\ d & -d & -1\end{pmatrix}$ a **correct** first step		B1
	$\frac{1}{-d-1}\begin{pmatrix}1 & -1 & d\\ -2-d & 1 & -d\\ 1 & -1 & -1\end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible** A1: Fully correct inverse	M1 A1 A1
			(5)
			Total 13

Question Number	Scheme		Marks
6(a)	$I_n = \int_0^4 \underline{x^{n-1}\times x(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$	M1: Obtains $x(16-x^2)^{\frac{1}{2}}$ prior to integration A1: Correct underlined expression (can be implied by their integration)	M1A1
	$I_n = \left[-\frac{1}{3}x^{n-1}(16-x^2)^{\frac{3}{2}}\right]_0^4 + \frac{n-1}{3}\int_0^4 x^{n-2}(16-x^2)^{\frac{3}{2}}\mathrm{d}x$		dM1
	dM1: Parts in the correct direction (Ignore limits)		
	$\therefore I_n = \frac{n-1}{3}\int_0^4 x^{n-2}(16-x^2)(16-x^2)^{\frac{1}{2}}\mathrm{d}x$		
	i.e. $I_n = \frac{16(n-1)}{3}I_{n-2} - \frac{n-1}{3}I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+\frac{n-1}{3}) = \frac{16(n-1)}{3}I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n = 16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*cso
			(6)
Way 2	$\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x = \int_0^4 x^n\frac{(16-x^2)}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x = \int_0^4 \frac{16x^n}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x - \int_0^4 \frac{x^{n+2}}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$		
	$= \int_0^4 16x^{n-1}\times x(16-x^2)^{-\frac{1}{2}}\mathrm{d}x - \int_0^4 x^{n+1}\times x(16-x^2)^{-\frac{1}{2}}\mathrm{d}x$		M1A1
	M1: Obtains $x(16-x^2)^{-\frac{1}{2}}$ prior to integration A1: Correct expressions		
	$=\left[-16x^{n-1}(16-x^2)^{\frac{1}{2}}\right]_0^4 + 16(n-1)\int_0^4 x^{n-2}(16-x^2)^{\frac{1}{2}}\mathrm{d}x$ $-(\left[-x^{n+1}(16-x^2)^{\frac{1}{2}}\right]_0^4 + (n+1)\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x)$		dM1
	dM1: Parts in the correct direction on both (Ignore limits)		
	$I_n = 16(n-1)I_{n-2} - (n+1)I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+n+1) = 16(n-1)I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n = 16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*
Way 3	$\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x = \int_0^4 x\times x^{n-1}\frac{(16-x^2)}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$	M1: Obtains $x(16-x^2)^{-\frac{1}{2}}$ prior to integration A1: Correct expression	M1A1
	$=\left[-x^{n-1}(16-x^2)(16-x^2)^{\frac{1}{2}}\right]_0^4 + \int_0^4 (16(n-1)x^{n-2} - (n+1)x^n)(16-x^2)^{\frac{1}{2}}\mathrm{d}x$		dM1
	dM1: Parts in the correct direction (Ignore limits)		
	$I_n = 16(n-1)I_{n-2} - (n+1)I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+n+1) = 16(n-1)I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n = 16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*

Question Number	Scheme		Marks
(b)	$I_1 = \int_0^4 x\sqrt{(16-x^2)}\mathrm{d}x = \left[-\frac{1}{3}(16-x^2)^{\frac{3}{2}}\right]_0^4 = \frac{64}{3}$	M1: Correct integration to find I_1 A1: $\frac{64}{3}$ or equivalent (May be implied by a later work – they are not asked explicitly for I_1)	M1 A1
	$\frac{64}{3}$ must come from correct work		
	Using $x = 4\sin\theta$: $I_1 = \int_0^{\frac{\pi}{2}} 4\sin\theta\sqrt{(16-16\sin^2\theta)}\,4\cos\theta\mathrm{d}\theta = \int_0^{\frac{\pi}{2}} 64\sin\theta\cos^2\theta\,\mathrm{d}\theta$ $= \left[-\frac{64}{3}\cos^3\theta\right]_0^{\frac{\pi}{2}}$ M1: A <u>complete</u> substitution and attempt to substitute <u>changed</u> limits A1: $\frac{64}{3}$ or equivalent		
	$I_5 = \frac{64}{7}I_3,\ I_3 = \frac{32}{5}I_1$	Applies to apply reduction formula twice. First M1 for I_5 in terms of I_3, second M1 for I_3 in terms of I_1 (Can be implied)	M1, M1
	$I_5 = \frac{131072}{105}$	Any <u>**exact**</u> equivalent (Depends on all previous marks having been scored)	A1
			(5)
			Total 11

Question Number	Scheme		Marks
7(a)	$(\frac{dx}{d\theta} = -a\sin\theta$ and $\frac{dy}{d\theta} = b\cos\theta)$ so $\frac{dy}{dx} = \frac{b\cos\theta}{-a\sin\theta}$		M1 A1
	M1: Differentiates both *x* and *y* and divides correctly A1: Fully correct derivative		
	Alternative: M1: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \Rightarrow y' = -\frac{b^2x}{a^2y} = -\frac{b^2a\cos\theta}{a^2b\sin\theta}$ Differentiates implicitly **and** substitutes for *x* and *y* A1: $= -\frac{b\cos\theta}{a\sin\theta}$		
	Normal has gradient $\frac{a\sin\theta}{b\cos\theta}$ or $\frac{a^2y}{b^2x}$	Correct perpendicular gradient rule	M1
	$(y - b\sin\theta) = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$	Correct straight line method using a „changed" gradient which is a function of θ	M1
	If $y = mx + c$ is used need to find *c* for M1		
	$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$ *		A1
	Fully correct completion to printed answer		
			(5)
(b)	$x = \frac{(a^2 - b^2)\cos\theta}{a}$	Allow un-simplified	B1
	$y = -\frac{(a^2 - b^2)\sin\theta}{b}$	Allow un-simplified	B1
	$\left(= \frac{1}{2}\frac{(a^2 - b^2)^2\cos\theta\sin\theta}{ab}\right)$	$= \frac{1}{4}\frac{(a^2 - b^2)^2}{ab}\sin 2\theta$	M1A1
	M1: Area of triangle is $\frac{1}{2}$"OA"×"OB" and uses double angle formula correctly **A1: Correct expression for the area (must be positive)**		
			(4)
(c)	Maximum area when $\sin 2\theta = 1$ so $\theta = \frac{\pi}{4}$ or 45	Correct value for θ (may be implied by correct coordinates)	B1
	So the point *P* is at $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ oe $\left(a\cos\frac{\pi}{4}, b\sin\frac{\pi}{4}\right)$ scores B1M1A0	M1: Substitutes their value of θ where $0 < \theta < \frac{\pi}{2}$ or $0 < \theta < 90$ into their parametric coordinates A1: Correct exact coordinates	M1 A1
	Mark part (c) independently		
			(3)
			Total 12

Question Number	Scheme		Marks
8(a)	$(\mathbf{6i}+\mathbf{2j}+\mathbf{12k}).(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})=34$	Attempt scalar product	M1
	$\left\lvert\dfrac{(\mathbf{6i}+\mathbf{2j}+\mathbf{12k}).(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})-5}{\sqrt{3^2+4^2+2^2}}\right\rvert$	Use of correct formula	M1
	$\sqrt{29}$ (not $-\sqrt{29}$)	Correct distance (Allow $29/\sqrt{29}$)	A1
			(3)
(a) Way 2	$\mathbf{r}=(\mathbf{6i}+\mathbf{2j}+\mathbf{12k})+\lambda(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})$ $\therefore\ 6+3\lambda\ \ 3+\ \ 2-4\lambda\ \ -4\ +\ 12+2\lambda\ \ 2=5$		M1
	Substitutes the parametric coordinates of the line through (6, 2, 12) perpendicular to the plane into the cartesian equation.		
	$\lambda=-1\Rightarrow\ 3,6,10$ or $-3\mathbf{i}+4\mathbf{j}-2\mathbf{k}$	Solves for λ to obtain the required point or vector.	M1
	$\sqrt{29}$	Correct distance	A1
(a) Way 3	**Parallel plane** containing (6, 2, 12) is $\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})=34$ $\Rightarrow\dfrac{\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})}{\sqrt{29}}=\dfrac{34}{\sqrt{29}}$	Origin to this plane is $\dfrac{34}{\sqrt{29}}$	M1
	$\Rightarrow\dfrac{\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})}{\sqrt{29}}=\dfrac{5}{\sqrt{29}}$	Origin to plane is $\dfrac{5}{\sqrt{29}}$	M1
	$\dfrac{34}{\sqrt{29}}-\dfrac{5}{\sqrt{29}}=\sqrt{29}$	Correct distance	A1
(b) **For a cross product, if the method is unclear, 2 out of 3 components should be correct for M1**	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&1&5\\1&-1&-2\end{vmatrix}=\begin{pmatrix}3\\9\\-3\end{pmatrix}$	M1: Attempts $(\mathbf{2i}+\mathbf{1j}+\mathbf{5k})\times(\mathbf{i}-\mathbf{j}-\mathbf{2k})$ A1: Any multiple of $\mathbf{i}+\mathbf{3j}-\mathbf{k}$	M1A1
	$(\cos\theta)=\dfrac{(\mathbf{3i}-\mathbf{4j}+\mathbf{2k}).(\mathbf{i}+\mathbf{3j}-\mathbf{k})}{\sqrt{3^2+4^2+2^2}\sqrt{1^2+3^2+1^2}}\quad\left(=\dfrac{-11}{\sqrt{29}\sqrt{11}}\right)$		M1
	Attempts scalar product of normal vectors including magnitudes		
	52	Obtains angle using arccos (dependent on previous M1)	dM1 A1
	Do not isw and mark the final answer e.g. 90 – 52 = 38 loses the A1		(5)
(c)	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&3&-1\\3&-4&2\end{vmatrix}=\begin{pmatrix}2\\-5\\-13\end{pmatrix}$	M1: Attempt cross product of normal vectors A1: Correct vector	M1A1
	$x=0:(0,\ \frac{5}{2},\ \frac{15}{2}),\quad y=0:(1,\ 0,\ 1),\quad z=0:(\frac{15}{13},\ \frac{-5}{13},\ 0)$		M1A1
	M1: Valid attempt at a point on both planes. A1: Correct coordinates **May use way 3 to find a point on the line**		
	$\mathbf{r}\times(\mathbf{-2i}+\mathbf{5j}+\mathbf{13k})=-\mathbf{5i}-\mathbf{15j}+\mathbf{5k}$	M1: $\mathbf{r}\times$ dir = pos.vector $\times$ dir **(This way round)** A1: Correct equation	M1A1
			(6)

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(c) **Way 2**	"$x + 3y - z = 0$" and $3x - 4y + 2z = 5$ uses their cartesian form of and eliminate x, or y or z and substitutes back to obtain two of the variables in terms of the third		M1
	$(x = 1-\frac{2}{5}y$ and $z = 1+\frac{13}{5}y)$ or $(y = \frac{5z-5}{13}$ and $x = \frac{15-2z}{13})$ or $(y = \frac{5-5x}{2}$ and $z = \frac{15-13x}{2})$		A1
	Cartesian Equations: $x = \dfrac{y-\frac{5}{2}}{-\frac{5}{2}} = \dfrac{z-\frac{15}{2}}{-\frac{13}{2}}$ or $\dfrac{x-1}{-\frac{2}{5}} = y = \dfrac{z-1}{\frac{13}{5}}$ or $\dfrac{x-\frac{15}{13}}{-\frac{2}{13}} = \dfrac{y+\frac{5}{13}}{\frac{5}{13}} = z$		
	Points and Directions: Direction can be any multiple $(0,\frac{5}{2},\frac{15}{2}), \mathbf{i}-\frac{5}{2}\mathbf{j}-\frac{13}{2}\mathbf{k}$ or $(1,0,1), -\frac{2}{5}\mathbf{i}+\mathbf{j}+\frac{13}{5}\mathbf{k}$ or $(\frac{15}{13},-\frac{5}{13},0), -\frac{2}{13}\mathbf{i}+\frac{5}{13}\mathbf{j}+\mathbf{k}$		M1 A1
	M1:Uses their Cartesian equations correctly to obtain a point and direction A1: Correct point and direction – it may not be clear which is which – i.e. look for the correct numbers either as points or vectors		
	Equation of line in required form: e.g. $\mathbf{r}\times(\mathbf{-2i}+\mathbf{5j}+\mathbf{13k}) = -\mathbf{5i}-\mathbf{15j}+\mathbf{5k}$ Or Equivalent		M1 A1
			(6)
			Total 14
(c) **Way 3**	$\begin{pmatrix}2\lambda+\mu\\ \lambda-\mu\\ 5\lambda-2\mu\end{pmatrix}.\begin{pmatrix}3\\ -4\\ 2\end{pmatrix} = 5 \Rightarrow 12\lambda+3\mu = 5$	M1: Substitutes parametric form of $\prod_2$ into the vector equation of $\prod_1$ A1: Correct equation	M1A1
	$\mu = \frac{5}{3}, \lambda = 0$ gives $(\frac{5}{3}, -\frac{5}{3}, \frac{10}{3})$ $\mu = 0, \lambda = \frac{5}{12}$ gives $(\frac{5}{6}, \frac{5}{12}, \frac{25}{12})$ Direction $\begin{pmatrix}-2\\ 5\\ 13\end{pmatrix}$	M1: Finds 2 points and direction A1: Correct coordinates and direction	M1A1
	Equation of line in required form: e.g. $\mathbf{r}\times(\mathbf{-2i}+\mathbf{5j}+\mathbf{13k}) = -\mathbf{5i}-\mathbf{15j}+\mathbf{5k}$ Or Equivalent		M1A1
	Do not allow 'mixed' methods – mark the best single attempt		
	NB for checking, a general point on the line will be of the form: **$(1 - 2\lambda, 5\lambda, 1 + 13\lambda)$**		